Acoustics of Resonators C.C. Banta

BACKGROUND

Christopher C. Banta has been working with the musical acoustics phenomena since 1968. His work consisted of loudspeaker systems design and pipe organ maintenance. While in the Navy, he worked with and studied radio communications. In 1973, he studied music theory at Pasadena City College, then transferred to California Institute of the Arts where he was granted a new category "Music—Special Studies". This was due to undertakings such as the constructions of Bass and Contra-Bass Marimbas and Pipe Organs. From Cal-Arts he went to James B. Lansing Sound, Inc. as a Quality Assurance Inspector. In 1976, he started a small loudspeaker business which eventually developed into "C. C. BANTA Creative Percussion Company". To this day, C. C. BANTA has provided bass marimbas and services to percussionists like: Emil Richards (Mr. World of Percussion), Carl Rigoli, Joe Porcaro, Steve Traugh (Supercussion), John Bergamo (Cal-Arts Percussion Ensemble), to Paramount Studios for recording of "SHO-GUN", and for commercial jingles in Chicago. Instruments have even gone to Japan.

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RESONATORS

Part 1 "Types and How They Work"

by: Christopher C. Banta

In general, all non-amplifying musical instruments require the use of acoustical resonation to communicate their sound into the environment. That is, an acoustic device or component to physically amplify or augment the movement of the original sound source which is usually a vibration of sorts.

If we take a marimba bar, perhaps a tenor "C" which is approximately 17" long by 2¼" wide and pinch it width-wise at the node points, then strike it with a mallet, the bar will vibrate 130.8 cycles per second (Hz). If you hold this bar an arm's length away from your ears, it will be very difficult to hear the fundamental pitch. At best, you may hear some overtones. As you bring the bar closer to your ear, the fundamental becomes more noticeable. Of course, if the bar is right next to your ear the fundamental is quite loud and substantial with every strike of the mallet. However, it is impractical to have *all* the bars of a standard marimba suspended about your ears and further more trying to play this sort of arrangement.

The problem is, the bar's back-and-forth movement creates a cancellation. That is, one side of the bar in movement cancels the other side (Fig. 1). This is because one side is always 180 degrees out of phase with the other. In this condition one side cannot couple or transmit its energy to the air properly. The ear has to be able to interpret this disturbance and rate of movement as audible sound. Since there is no practical means to isolate the two sides from each other, we have to create a secondary sound source triggered by the marimba bar which is the primary sound source. This is where the resonator is most useful. A resonator is defined as "a hollow body, such as a chamber or column, that responds to the vibrations of a given frequency."

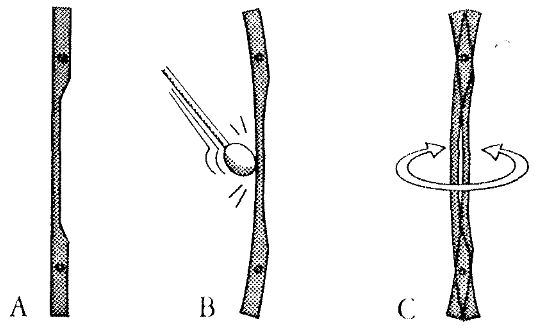

Fig. 1 A) Bar at rest. B) Initial mallet strike. C) Back & forth movement, each side cancelling the other.

TUBE RESONATORS

There are several types of resonant apparatuses, but for this topic we will discuss specifically two kinds. They are:

1. Tube or Column resonator
2. Helmholtz resonator

The tube type resonator can simply be defined as "a hollow column whose diameter is consistent throughout its length" (Fig. 2). This applies to square, rectangular, oval, triangular, even hexagonal and octangular cross-sections.

There are two types of tube resonators:

A. ½ wavelength
B. ¼ wavelength

The ½ wavelength (symbol for wavelength is λ) resonator is a tube that is open at both ends (Fig. 2A) and whose length can be determined by the formula:

$$L = \frac{1129/f}{2}$$

Where: L = overall length of tube in feet.

f = resonant frequency in Hertz.
1129 = speed of sound in ft/sec @ 70 degrees F.

A ½ λ resonator contains a full series of overtones in addition to its fundamental. If the fundamental is 100Hz, the next overtone is 200Hz, then 300Hz, 400Hz, 500Hz, and so forth. The ¼ λ resonator is a tube that is closed at one end and open at the other (Fig. 2B). Its length can be determined by the formula:

$$L = \frac{1129/f}{4}$$

Where: same parameters apply.

A ¼ λ resonator contains only the odd harmonics in addition to its fundamental (*100Hz, 300Hz, 500Hz, 700Hz, etc.*).

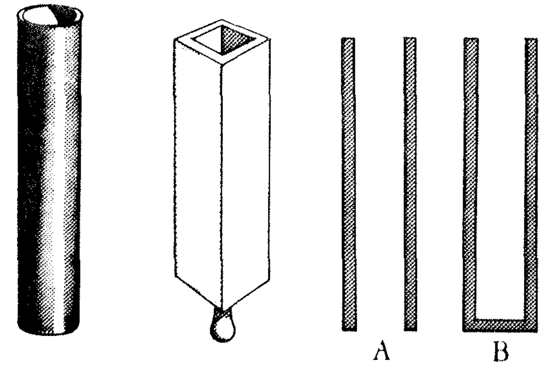

Fig. 2 TUBE RESONATORS
A) 1/2 wavelength. B) 1/4 wavelength.

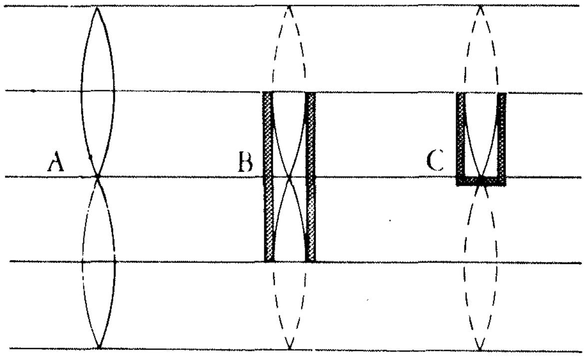

Fig. 3 A) Full wavelength of a frequency. B) 1/2 wavelength resonator. C) 1/4 wavelength resonator.

It will become clear the differences between both types of tube resonators. ½ λ resonators have to be twice the length as ¼ λ resonators in order to produce the same frequency (Fig. 3). This does not mean that one is any better than the other, but in the interest of saving space, the ¼ λ resonator is preferrable. Percussion instruments can be made at playable heights using half the amount of materials. On the other hand, ½ λ, open-ended resonators more approximate the true wavelength of a frequency.

THE RESONATOR AND ITS PITCH

If you take a cylinder (closed ended or not) and hold it in one hand then slap or pop the open end with the other hand, you will hear a definite pitch emanate from the opening(s). This is the resonant frequency or resonance of the tube. Resonance occurs when the natural frequency of a body (mass) is set into motion by a reinforcing frequency or vibration (energy). At this resonant frequency, a small amount of energy is required to move the mass. This is not to be confused with "sympathetic vibration." A resonator tuned to a specific frequency can only resonate at that frequency. But, depending on the resonator, there are certain related harmonic partials that can also be excited. Anyhow, what the ear hears is the column of air inside the tube that has been disturbed and set into motion by the slap. It transfers this energy to the surrounding air which reaches the ear. The length determines what frequency is sounded, not the slap.

THE COUPLED SYSTEM

Let us take our tenor "C" marimba bar (130.8Hz) and a 25.9" long closed-ended tube—based upon the formula:

$$L = \frac{1129/f}{4}$$

$$L = \frac{1129/130.8}{4}$$

$$L = \frac{8.63}{4}$$

L = 2.16 ft. times 12" = 25.9"

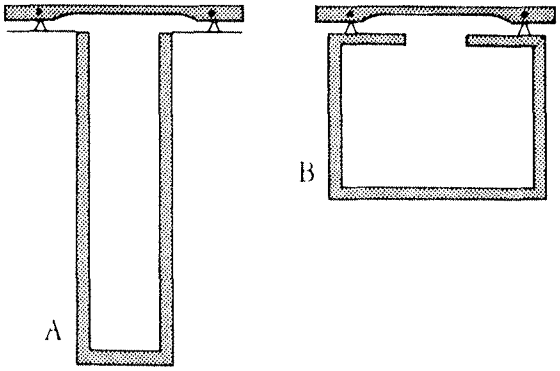

Fig. 4 Bars suspended over resonators.

and support the bar over the opening of the tube (Fig. 4A). Now strike the bar with a mallet. The resonance heard will be quite loud and sustaining. What we have created is a well coupled system.

Closer analysis shows that both resonator and bar are tuned to the exact same frequency. So when the bar is struck, it sends a surge through the resonator opening. This wave travels down the complete length of the tube until it hits bottom, then bounces or reflects, and returns back to the opening. As the wave approaches the opening, the bar will be on its upswing. All through the cycle, both bar movement and column movement remain in perfect phase with each other which is necessary to maintain the coupling. When the motion of the bar comes back down again due to its stiffness, mass, and by trying to reach a point of equilibrium, it creates another surge that enters the tube,thus repeating the cycle but in rapid succession.

The reason the tone or resonance is so full is because instead of a slap to set the column into movement there is now a properly matched frequency generator (bar) whose movement corresponds to the length/frequency of the air column and can be set into motion very efficiently. Unfortunately, each successive swing of the bar has less amplitude than its previous one and will eventually zero. Also, due to the frictional and radiational losses in the tube, the sound will begin to die out.

IMPROPER COUPLING

If the bar vibrating its normal rate is placed over a resonator *longer* than its normal coupling frequency, the motion of the bar will return faster than the wave in the resonator has a chance to catch up to it. On the other hand, if the column is *shorter* the wave inside the tube will reach the opening faster than the bar's movement. Both of these conditions result in improper coupling with little or no resonance occurring.

SIZE AND LENGTH

Sizewise, the one unfortunate aspect of lower notes is that the lower the frequency, the longer the resonator. Although many percussion instrument builders compromise on the sizing of bars for ease of the performer (with the exception of the late Harry Partch), you cannot fool the wavelength. A 50Hz pitch has a 22½ foot long wavelength. If it were shorter it would be higher in pitch. Both the open-ended and closed-ended resonator have to match a proportional segment of the wavelength that corresponds to the desired pitch (*see Fig. 3*).

Scaling is a term that pipe organ builders give to their pipes. Some pipes have very wide cross-sectional dimensions in proportion to their lengths while others are very skinny and tall. The purpose is to augment or defeat the amount of overtones that occur within the column. Generally, wide-scaled pipes have not only an increase in amplitude (loudness), but more fundamental with less overtone content. Thin pipes which are abundant in overtones tend to have an attenuated fundamental. The same principle would apply to the tube resonator. Also, as a rule of thumb, the diameter or cross-sectional dimension of the tube should not be less than one-tenth its overall length.

HELMHOLTZ RESONATORS

The Helmholtz resonator is defined as "an interior connected to the exterior by means of an opening" or more specifically, a chamber or cavity whose internal dimensions are larger than its opening (Fig. 5A). It comes from the German Physicist, **Hermann Ludwig Ferdinand von Helmholtz** (1821-1894) who had originally conceived of special resonators for use in the analysis of specific frequencies. They were hollow spheres made of brass with two openings. A large round opening opposite a small opening atop a taper so that it could be inserted into the ear (Fig. 5B). He devised a complete set of resonators each tuned to a particular pitch. They had a variety of uses, but a single unit could pick out a frequency while a full orchestra was performing or could be used in checking the tuning of musical instruments.

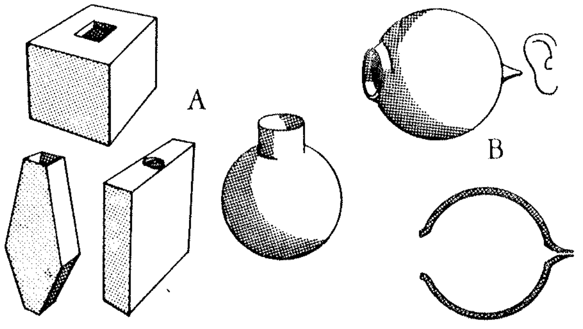

Fig. 5 A) Types of Helmholtz resonators. B) Original Helmholtz resonator used for analyzing specific frequencies.

The spherical resonator is the most efficient type of Helmholtz resonator because all internal surface dimensions are equi-distant. A cubical shape would be the next most efficient type. Unfortunately, these shapes are not the most practical for laying out musical notes and scales. So, there have been certain compromises in their geometry and appearance. John Brock, who built bass marimbas in the 1940's, used long skinny but very wide rectangles with thin walls so that the low frequency notes could be fitted within a playable proximity to each other. The Guatemalan marimbas use large truncated bi-pyramidal shaped resonators (Fig. 6) that tend to *fan* out at the lower register of the instrument because bass notes require larger internal volumes. African gourd marimbas took on a *concave* appearance in order to accommodate all the resonators suspended beneath the bars. Sometimes there are certain problems associated with odd-shaped Helmholtz resonators. They could have additional resonant frequencies other than just the fundamental which may not necessarily follow any harmonic relationship (Fig. 7).

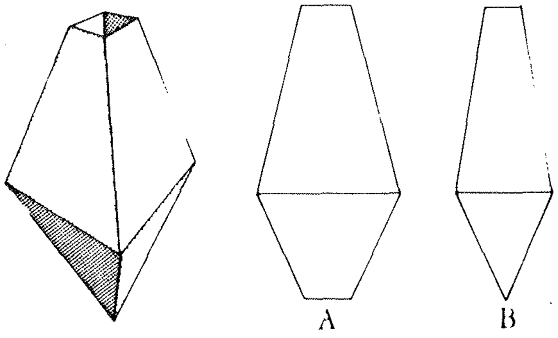

Fig. 6 Guatemalan resonator using Helmholtz principle. A) Side view. B) Front view

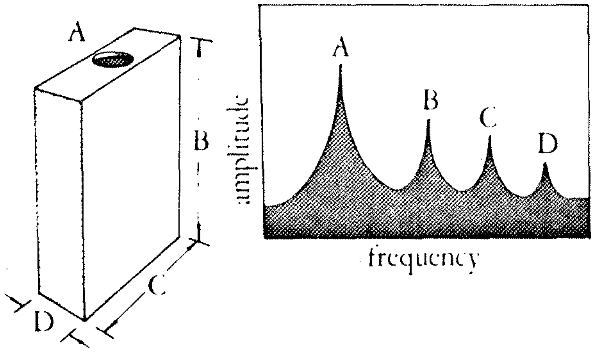

Fig. 7 Rectangular Helmholtz resonator response showing; A) Fundamental resonance of box. B) First major dimension. C) Next dimension. D) Minor dimension.

Helmholtz resonators are much more complex in nature than tube types, therefore accurate analysis requires much time and sophisticated test equipment. Formulas depend on several parameters to obtain such analysis. However, there is a relatively simple formula that can be used to figure out the resonant frequency of a cavity and with reasonably good results. It is:

$$f = 1856.1 \sqrt[4]{\frac{A}{V^2}}$$

Where:
f = resonant frequency in Hertz.
A = area of opening in Sq. In.
V = total internal volume in Cu. In.
1856.1 = a constant

This formula can also be transposed to figure out Opening area and Volume. They are:

Area

$$A = \left(\frac{f}{1856.1}\right)\sqrt[4]{V^2}$$

Volume

$$V = \left(\frac{1856.1}{f}\right)^2$$

VOLUME AND SIZE

As with the tube resonator, frequency is a function of size. Basically, large resonators have lower resonant frequencies than smaller resonators. Also, it should be noted that the opening plays a crucial role in the relationship. The smaller the opening per given volume the lower the resonating frequency. Conversely, the larger the opening per given volume, the higher the resonant frequency (Fig. 8). Size also has a direct effect on amplitude. If you have a large and a small resonator both with the same resonant frequency, the larger will be louder. This is because a larger volume can transfer more energy which will move more air, thus increasing the amplitude.

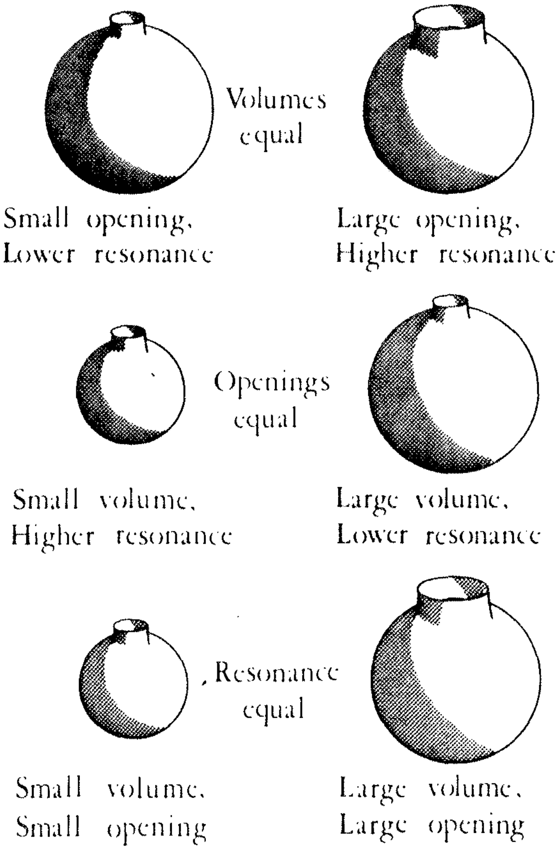

Fig. 8 Basic volume, opening, & resonance relationships.

SOUND OR NOT FROM THE HELMHOLTZ

Just like the tube, you can slap the opening or side of a Helmholtz resonator and hear its resonant frequency. If we place a tuned marimba bar over a properly coupled resonator (Fig. 4B) and strike it, a full tone will come forth. However, it should be noted that the Helmholtz resonator can actually "suck" the frequency it is tuned to out of the air! This phenomenon can be used to help the acoustics in studios and concert halls. Wools and fiberglass can control high frequency resonances but are less effective at low frequencies. Therefore, large resonators tuned to specific frequencies can help suppress certain undesirable standing waves. But, to have the resonance respond to the fullest, the sound source has to be directly over the opening. As you move the sound source away from the resonator, it will actually attenuate in amplitude. This is because the resonator will not allow any sound that approaches its resonant frequency to exist near it. But, if you cover the resonator opening, the level of the sound source will be restored (Fig. 9).

RIGIDITY

It is very important for the Helmholtz resonator to have rigid walls. Depending on the thickness and strength of the materials being used, the energy brought into the cavity could be absorbed by the walls. This would certainly take away from the efficiency of the cavity. Thin walls will work if properly braced and made to not *breathe* with the constant input energy. But, thick non-porous walls are optimum.

This concludes the first of two parts. Part two will discuss RESONATORS—"How to Tune and Measure Them." o

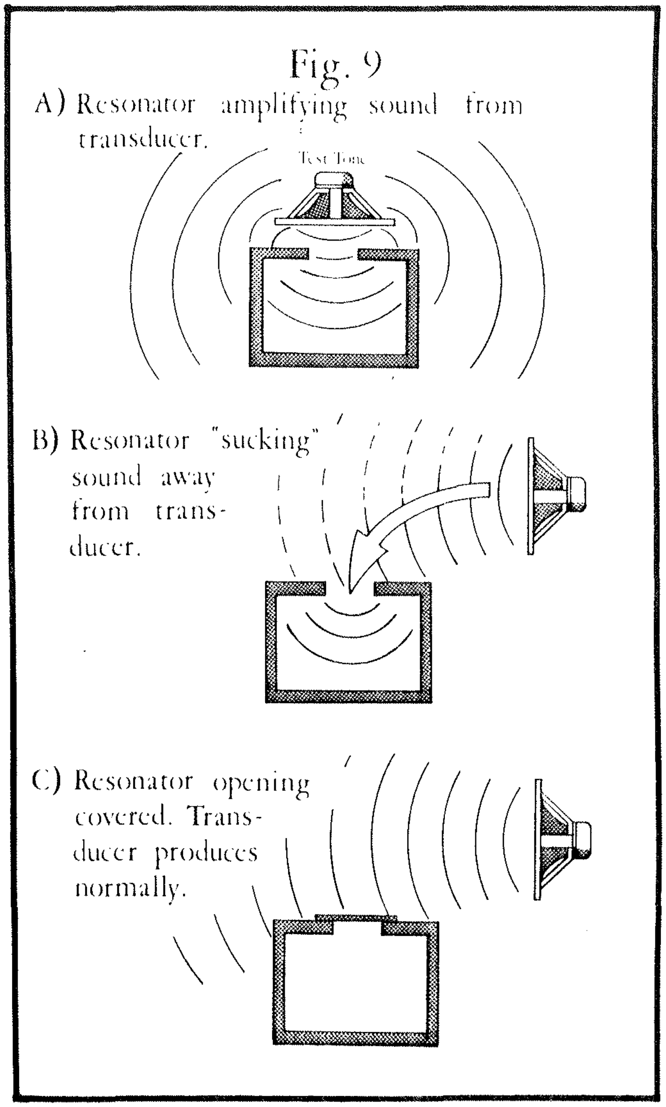

Fig. 9

A) Resonator amplifying sound from transducer.

B) Resonator "sucking" sound away from transducer.

C) Resonator opening covered. Transducer produces normally.

RESONATORS Part II C.C. Banta

RESONATORS
Part 2
How to Tune and Measure Them

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Acoustic resonators have a fundamental frequency in which only a small amount of energy is needed to excite or set the system into resonation. It is at this point the system exhibits its maximum amount of energy and amplitude. However, the external motivating force must have the exact same frequency as the system, otherwise the resonating system will not be set into motion.

Acoustic systems whether they are loudspeaker enclosures or resonators require some kind of tuning for optimum performance. In the case of the resonator, tuning will cause it to respond to a specific fundamental frequency. The result will:

1. Amplify a sound source when the source is located above the opening. (Similar to a marimba bar suspended over a resonator.)
2. Absorb any sound whose frequency matches the resonant frequency of the resonator.

The resonator actually performs both of these functions simultaneously, but for the sake of the musical instrument, we are interested only in the first result. So, what becomes important is the position and direction of the motivating sound source.

COLUMN OR TUBE RESONATORS

The tube type resonator represents an acoustic system whose wavelength is a *proportional segment* of the tube's length. In the tube, the resonating frequency is controlled by length not volume. The basic regulating factor for the tube is:

To lower pitch— increase length
To raise pitch—shorten length

This applies to all tubes, both $\frac{1}{2}\lambda$ and $\frac{1}{4}\lambda$ resonators. (See RESONATORS, Part 1, INTERVAL Summer 1981.) Since higher frequencies have shorter wavelengths, the same applies to tube resonators. The shorter the resonator, the higher the pitch. The longer the wavelength, the lower the pitch or slower the frequency. To halve a frequency of a tube, whether it is open at both ends or closed at only one end, the length must be doubled. To double the frequency, the length must be cut in half. This phenomenon lends itself very nicely for adjustments of the column length.

TUNING METHODS

The following are tuning methods that can be used to adjust the column length.

—To raise pitch (shorten column)
- Method 1. Slider (Fig. 1A). Tube must be longer than desired length.
- Method 2. Remove a portion of the tube. Too permanent.
- Method 3. Flare the opening. Applicable only to round tubes.
- Method 4. Stopped $\frac{1}{4}\lambda$ resonators: push stopper in (Fig. 5A).

—To lower pitch (lengthen column)
- Method 1. Slider (Fig. 1A). Tube must also be longer than desired length.
- Method 2. Physically add length.
- Method 3. Cover a portion of the opening (Fig. 1B).
- Method 4. Stopped $\frac{1}{4}\lambda$ resonators: pull stopper out (Fig. 5A).

All methods have a limited range which they cover. For instance, to significantly lower an existing frequency, a new length altogether would be necessary.

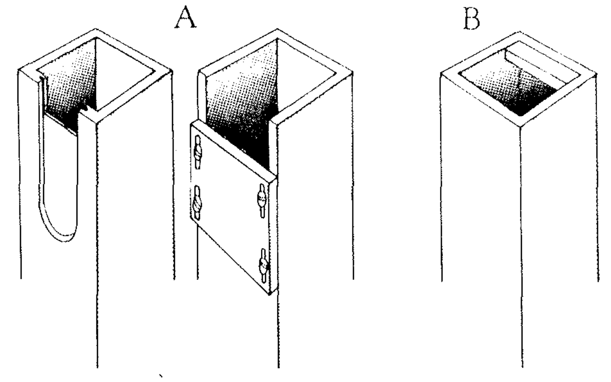

Fig. 1 Tube tuning slides

MITERING

Mitering is a method of bending or changing direction of the column so that it will conform to smaller spaces (Fig. 2). This is a practice pipe organ builders have done for centuries. In bass marimbas, the longer resonators would have to be mitered in order to keep the instrument at a playable height. Resonators could be neatly tucked underneath the bars. Mitering also has no effect on sound output or frequency, but abrupt corners tend to make travel of the internal wave less efficient.

HELMHOLTZ RESONATORS

It should be noted that the Helmholtz resonator represents an acoustic system that is *small* in comparison to its wavelength. The frequency of this type of system is controlled by volume, not by any single dimension such as length, width, or depth.

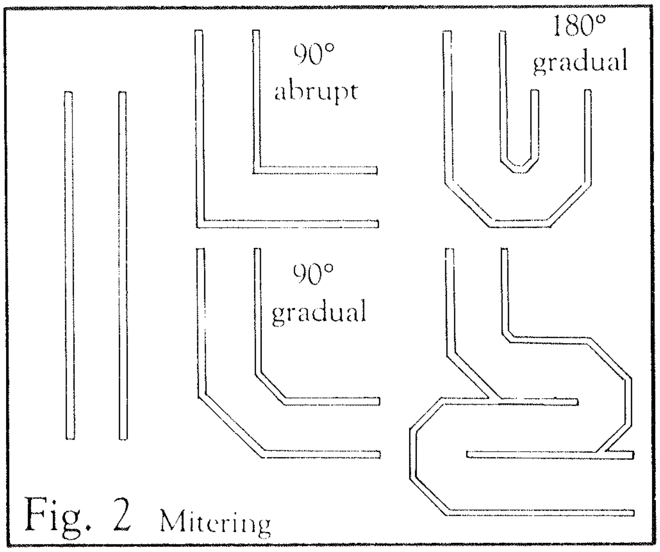

Fig. 2 Mitering

The basic regulating factor for the Helmholtz resonator is:

To lower pitch 1. Increase internal volume and/or
2. decrease opening area.
To raise pitch 1. Decrease internal volume and/or
2. increase opening area.

When an external sound source matches the resonator's frequency, it causes the air inside the cavity to compress and expand at a predetermined rate or frequency. This rate is controlled by a ratio of area to internal volume. An analogy can be used to describe the effect more clearly. A large bottle has more capacity than a small bottle and therefore holds more water. Both bottles require a certain time to be filled. With a steady stream of water, the large bottle will fill to capacity much slower than the small bottle to its capacity. At this point, the larger bottle can be said to have a lower frequency. The opening also determines the rate to which they fill and empty. A large opening can accept water faster thus filling to capacity quicker (high frequency). A smaller opening can't accept the water as fast and will take longer to fill (lower frequency). The same would hold true for emptying the bottles. Here is a situation where any combination can be employed. For example, a bottle with a large opening and a small capacity can fill and empty very rapidly, while a larger bottle with a small opening will take much longer.

Here are some rules to be noted:

To halve an existing frequency, you have to:
1. Decrease the opening area to one-fourth its original size.
or
2. Increase the internal volume by four times.

To double an existing frequency, you have to:
1. Increase the opening area four times.
or
2. Decrease the internal volume to one-fourth its original size.

AMPLITUDE

If we have a cavity with an internal volume of 1034 cu. in. with an opening area of nine sq. in., the resonant frequency will be approximately 100Hz. This represents the optimum resonant frequency you can get from this opening-to-volume ratio. To increase the loudness or amplitude you need a resonator with a larger opening and a proportionally larger internal volume. Now, there is a certain degree to which you can vary the resonant frequency. If you want to lower the resonance from 100Hz down to 95Hz, this size resonator can probably accommodate it without too much loss of loudness. But, to go down to around 70Hz would be out of the question. There would be a significant loss of amplitude thus resulting in attenuated output no matter what size your primary sound source is (Fig. 3). It is this reason alone why the Helmholtz resonator has only a limited range or latitude to which it can respond faithfully to a frequency.

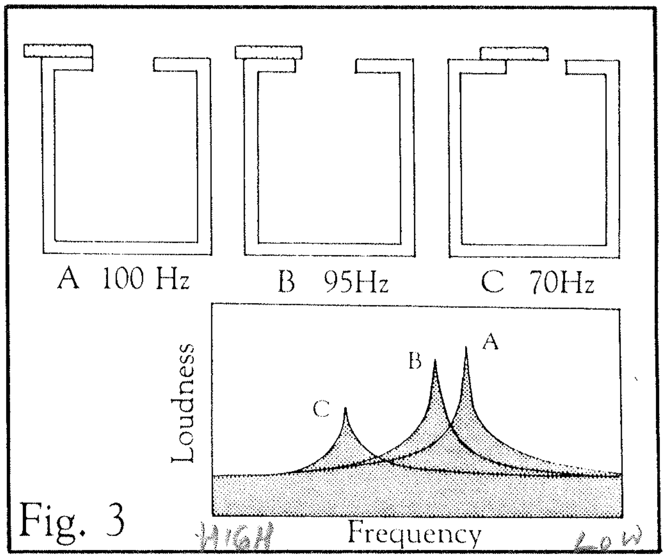

Fig. 3

METHODS OF TUNING

Method 1: Control of the opening

A simple sliding mechanism can be mounted on top of the resonator, partially covering the opening (Fig. 4). The hole should be about 30% longer or larger. Now, by sliding the cover over the opening a little bit at a time, you will hear a slight lowering of the pitch. Slap the side of the resonator and hold the opening close to your ear. The resonant frequency will come forth. Do not hold the opening too close, otherwise your head will create additional flattening of the pitch.

Method 2: Control of the volume

What is necessary in this method is to have control over the displacement of volume. One way would be to build a resonator with a duct and stopper (Fig. 5B). The stopper can be pushed in or pulled out, causing a displacement of volume, thus raising or lowering the pitch. Another way that can help an oversized resonator is by throwing small pieces or blocks of wood into the resonator. You will create displacement of the internal volume, but of course, too much wood will raise the resonance too high.

Method 1 is the primary way to tune a Helmholtz resonator. Method 2 is secondary and should be done as a last resort.

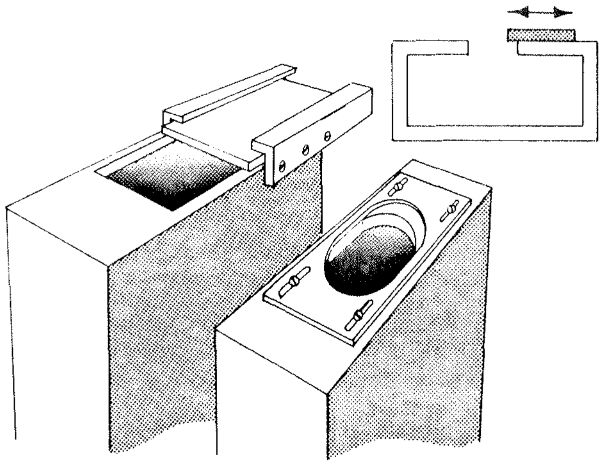

Fig. 4 Helmholtz tuning slides

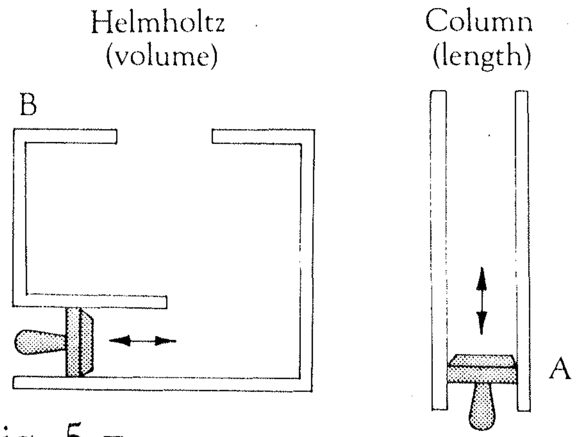

Fig. 5 Tuning stoppers

MEASURING WHAT YOU'VE TUNED

The ear is a remarkable mechanism for hearing distinct pitches and tones, but unfortunately it is not as accurate as a Strobe tuner. It is extremely difficult to remember *exact* frequencies, especially with lower tones. This is where electronic test equipment is essential. Test equipment is not complicated and can become an invaluable tool due to its ability to measure the results with extreme accuracy.

There are four pieces of equipment that can be used for testing purposes. They are:

1. Audio oscillator—generates a continuously variable test tone from 20Hz to 20KHz.
2. AC Voltmeter—a voltage measuring device.
3. Frequency counter—used to actually count the cycles per second, then display the quantity.
4. Strobe tuner—an accurate tuning device.

Some peripheral equipment will also be necessary.

1. Audio amplifier—a voltage boosting device.
2. Loudspeaker—may be open or enclosed, but should have a frequency response from 30Hz to 1KHz.
3. Microphone—preferably the small lavalier variety which offers little or no displacement when inserted into the resonator.

Fig. 6A shows a simple method of verifying pitch (not frequency) using a Strobe tuning device. The stroboscopic lines on the spinning disc represent the exact pitch when standing perfectly still. This works in conjunction with a dial that indicates the notes of the scale and another dial that gives inbetween steps that are measured in cents (hundredths of a semitone). A microphone is connected to the strobe tuner and placed near the opening of the resonator. Now, the side or bottom of the resonator can be slapped, thus causing a momentary tone burst or boom tone to emanate from the opening. The strobe will show an instantaneous indication until the tone burst dies out. The stroboscopic lines will move to the right or to the left if the resonator is sharp or flat from desired pitch. Tuning adjustment will correct this.

Fig. 6

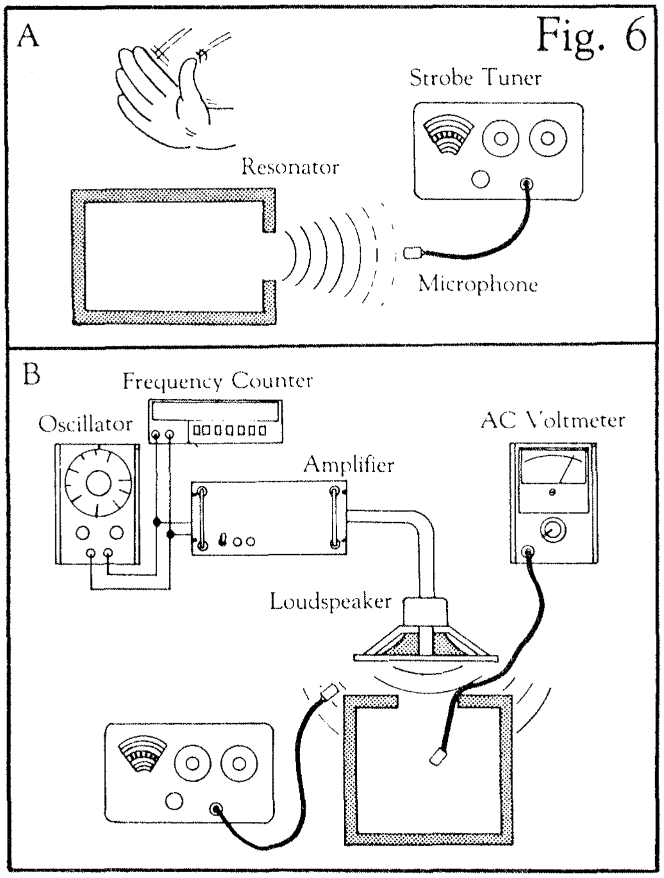

Fig. 6B shows a more accurate method for checking the tuning and frequency of the resonator. The block diagram indicates how the test equipment is connected and this is basically how it works: The *oscillator* produces a sine wave test tone which is fed into the *amplifier*. The amplifier drives the *loudspeaker* which audibly reproduces the test tone and conveys this energy into the opening of the *resonator*. The *frequency counter* gives better resolution of the dial markings on the oscillator and is necessary for frequency measurements. The *microphone* is placed inside the resonator and then connected to the *AC voltmeter* which visually indicates an increase or decrease in voltage. The increase of voltage is the increase of energy or pressure picked up by the microphone when the resonator nears its resonant frequency. The *strobe tuner* is used to verify the tuning pitch of the resonator, but only when the needle on the AC voltmeter has reached its peak. Do not peg the scale.

During the tuning process, both types of resonators will have to be constantly monitored for controlled results.

A word of CAUTION! The energy *inside* the resonator is extremely powerful and can produce a very dangerous sound level to a small microphone, especially when approaching the point of resonance. Keep the level of the loudspeaker to a minimum or very soft volume at best. Remember, resonators are very efficient at their resonant frequencies and not much energy is required to excite them.

Parts 1 and 2 on RESONATORS should give the experimenter a foothold on this interesting field of musical acoustics. There will be many trials and errors encountered, but this can only aid in the proper understanding of resonators and how they work.